Is Mathematics Connected to Religion?

by Stanisław Krajewski

The penultimate version of the chapter to appear in: Sriraman, Bharat (ed) *Handbook of the History and Philosophy of Mathematical Practice*. Springer, Cham, 2022.

Already available online at DOI https://doi.org/10.1007/978-3-030-19071-2_77-1

Abstract

Modern mathematics has been shaped by the process of secularization of science. Yet even some present-day mathematicians use religious terms behind the (mathematical) scenes. How essential this is remains debatable. Before modernity, everything, including mathematics, was perceived from a religious perspective. Deeper connections existed for Pythagoreans, and nowadays there is a revival of Pythagoreanism. Mathematics was used by medieval theologians, and even the founders of modern science – for instance, Newton, Leibniz, Boole, Cantor – referred to religion. From the present perspective, those theological aspects seem mostly irrelevant. The concept of actual infinity had a fundamentally religious meaning before the nineteenth century. Now, only a few see mathematical infinity as a notion common to mathematics and theology.

Religious origins of some mathematical concepts can be detected, for example, Indian origins of zero, possible Christian sources of the initiation of the algebraic approach, the creative function of naming applied to infinite entities. On the other hand, mathematical illustrations of theological concepts are possible, even though genuine mathematical models do not seem to be essential or testable. Generally, religious needs and motivations played a role in the development of mathematics, and mathematical metaphors helped theologians. To perceive this connection as relevant now requires sensitivity to the realm of the religious.

Keywords Philosophy of mathematics, Theological mathematics, God's mind, Religious metaphors, Religious inspirations, Mathematical models in theology, Infinite sets, God's point of view, Pythagoreans, Cantor

> ἀεὶ ὁ θεὸς γεωμετρεῖ
>
> God always geometrizes. (Attributed to Plato)

> Mathematica est inimicissima omnino theologiae.
>
> Mathematics is theology's greatest enemy of all. (Martin Luther)

> Das Leben der Götter ist Mathematik. … Reine Mathematik ist Religion.
>
> The life of the Gods is mathematics. … Pure mathematics is religion. (Novalis)

To consider relations of mathematics and religion we must assume we know what these two entities are. It is not easy to define mathematics but in the present Handbook familiarity with this field is presumed. What about religion, or rather religions, and theology, a faith-based rational study of religious issues? It seems hopeless to define religion. The best way to illustrate this difficulty is to quote Wilfred Cantwell Smith: “the question ‘Is Confucianism a religion?’ is one that the West has never been able to answer, and China never able to ask” (Smith 1964, p. 66). We have no choice but to assume familiarity with religious functioning and thinking about divine matters. The main reference points are the Abrahamic religions: Judaism, Christianity, and Islam; the religions that developed in India; and the legacy of Greek theological philosophy.

1 On the Approach to the Problem

While it may be true that everything is connected to everything else, mathematics– especially the practice of mathematics as we see it nowadays – seems unrelated, indeed totally opposite, to religion and theology. To modern eyes there exist no other two fields as unrelated as numbers and formal structures on the one hand, and God and divine matters on the other; or as calculating and proving on the one hand and praying and prostrating on the other. Yet to premodern eyes the connection was obviously there. The first task in the attempt to answer the question of whether mathematics is connected to religious matters should be to indicate the old understanding of the relationship and, as a part of it, to indicate those who were both: mathematicians and theologians or passionately religious men. This task can be fulfilled only partially, by giving a few examples (in Sect. 3), since a full presentation would be close to a comprehensive history of the entirety of human thought.

Modern mathematics is the result of a process of secularization of science in general and mathematics in particular. The process took place mainly in the nineteenth century. It would be a mistake, however, to assume that the establishment of modern branches of mathematics was always done in the secular spirit.

It is of special interest to note that even present-day mathematicians use religious terms. To say that mathematics is for us “a divine language” constitutes a standard, if certainly metaphoric, idiom. The presence of such links is illustrated in Sect. 2. The most important (allegedly) common concept is, of course, infinity. It is treated at the end of Sect. 3, but it is also present at almost every point since some reference to infinity appears in most cases where we can observe connections of the mathematical to the religious.

Among more specific connections that can link mathematics and religion are the allegedly religious origins of some principal mathematical concepts (Sect. 4) and attempts to apply mathematics in theology (Sect. 5).

Comprehensive treatment of the subject seems hardly possible at this stage of research. According to Ivor Grattan-Guinness, the “secularization of mathematics has led historians to ignore religious factors in the development of mathematics, and even more of mathematical elements in religion” (Grattan-Guinness 2009, p. 278). To be sure, there are many publications researching aspects of the problem. Noteworthy recent books include the comprehensive volume edited by Teun Koetsier and Luc Bergmans 2005, the collection on infinity edited by Michał Heller and Hugh Woodin 2011, special issue on mathematics of the journal *Theology and Science* vol. 9, No 1, 2011, edited by James Bradley, and the issue of *Studies in Logic, Grammar and Rhetoric* 44 (57), 2016, subtitled "Theology in Mathematics?", edited by Stanisław Krajewski and Kazimierz Trzęsicki. A more popular collection was edited by Lawrence and McCartney (2015). Book length studies include Albertson 2014 dealing with medieval history, Cohen 2007 treating Victorian Britain, and Graham and Kantor (2009), describing the role of Orthodox Christian heresy "Imiaslavie" (see Sect. 4).

1.1 What This Chapter Is Not

The present chapter attempts to avoid prejudgments. The approach is cautious: some connections of mathematics and religion are indicated but no claim is made that they are very deep or absolutely essential unless this can be demonstrated. It is important to distinguish this relatively inconclusive approach from the essentially faith-based position adopted by some authors, especially in the USA, who advocate the religious impact of mathematics or a religious dimension of it. While publications under the rubric "Christian mathematics" can include valuable works, they are all written with the presumption that mathematics is religiously relevant. And this assumption is a corollary to a more basic belief: everything is God's creation so we can perceive him everywhere, including in mathematics. This approach easily leads to doubtful claims. One can state that "the truth 2 + 2 = 4 has the attributes of God, such as omnipresence, eternity, omnipotence, and truthfulness. God spoke the world into existence" (Poythress 2015, p. 42). This characterization is not without interest, but it brings no insight when the conclusion is that "when we analyze what 2 + 2 = 4 really is, we find that arithmetic constantly confronts us with God himself, the Trinitarian God" (p. 25). In addition, "in thinking about arithmetic, we are thinking God's thoughts after him" (p. 25). The image behind all those statements is a theological assumption that used to be standard and normative two or more centuries ago but is totally unconvincing to most scientists today.

To mention a much more serious achievement, Bradley and Howell 2011 present a competent popularization of several mathematical topics. They argue that mathematics is fascinating and beautiful, the points which will be approved by every mathematician, and end each chapter with a statement like "Isaiah 1:18 asks us to reason with God, and the logical structure of mathematics gives us a way to reason effectively" (Bradley and Howell 2011, p. 138). The authors offer a general conclusion saying that "the Christian students would realize that the philosophy of mathematics and Christian faith share a number of interesting touch points. Indeed all things exist and have their being through Christ, including mathematical objects" (p. 217). This does not seem illuminating, for it is just presumed. That is why such books, even when they contain good presentations of mathematics, do not answer the question about the relation of mathematics to religion asked by unprejudiced inquirers.

Rather than trying "to redeem" mathematics, the approach adopted in this chapter is critical, assuming no religious starting point, but at the same time, it is open to the realm of the religious. It is neither assumed that everything must be according to religious convictions nor that nothing religious can be essentially tied to mathematical practice.

2 Mathematicians Using Religious Metaphors

In 1883, Georg Cantor expressed a strikingly modern opinion: the essence of pure mathematics is its freedom (Cantor 1932, p. 182). He wanted to defend the meaningfulness and significance of his pioneering work; he felt that his theory of actually infinite sets was consistent and compatible with the existing mathematics. This opinion is today absolutely dominant among mathematicians. Cantor and David Hilbert were among its first spokesmen. It was Hilbert, who in 1925, made the famous remark on behalf of mathematicians: "No one shall drive us out of the paradise which Cantor has created for us" (Hilbert 1926; Benacerraf and Putnam 1983, p. 191). Is the religious terminology he used only a convenient and universally understandable rhetorical device or does it express something substantial? Even if he just wanted to use a nice metaphor one can still ask why religious metaphors are particularly satisfactory when we talk about mathematics (Krajewski 2016). This question is naïve, perhaps too naïve, but the ease with which mathematicians utilize theological language must not be ignored. To be sure, this language does not appear inside proofs or definitions, but rather on the side, in informal speech when one wants to say why some topic is important or interesting, and what were one's motivations or associations. On such occasions it can be perfectly natural to talk about God, "God's standpoint," "God's mind." Occasionally traces of this way of talking can be retained in the "official" text. Thus, we can talk about performing infinitely many acts (for instance, acts of choice of an element from a set) as if we had an infinite, "divine" mind; we can refer to a complete knowledge (for instance, taking the set of all sentences true in a given interpretation) as if we were actually omniscient. We can also refer to paradise in Hilbert's sense.

One might say that all such figurative utterances using, directly or indirectly, theological terms are irrelevant and should be ignored in reflection about the nature of mathematics; they are mere chatting, present around mathematics, but not part of it. This loose conversation is, however, part of mathematics, says Reuben Hersh, who asks us to consider seriously the fact that mathematics, like any other area of human activity (as analyzed by Erving Goffman), has a front and a back, the chamber and the kitchen. The back is of no small importance since the product is made there. The guests or customers enter the front door but the professionals use the back door. Cooks do not show the patrons of their restaurant how the meals are prepared. The same can be said about mathematicians, and for this reason, its mythology reigns supreme. It includes, says Hersh, such "myths" as the unity of mathematics, its objectivity, universality, and certainty (due to mathematical proofs). Hersh is not claiming that those features are false. He reminds us, however, that each has been questioned by someone who knows mathematics from the perspective of its kitchen. Real mathematics is fragmented, it relies on esthetic criteria, which are subjective, proofs can be highly incomplete, and some of them have been understood in their entirety by nobody. Seen from the back mathematics is "fragmentary, informal, intuitive, tentative. We try this or that, we say 'maybe' or 'it looks like'" (Hersh 1991, p. 128; also in 2006). And we do borrow from the religious language. Perhaps the most telling example is the joking contention of Paul Erdös, the famous author of some 1500 mathematical papers (more than anyone else), according to which there exists "The Book" in which God (who else?) has gathered the most elegant proofs of mathematical theorems. Erdös was far from standard religiosity, but he reportedly said, "You don't have to believe in God, but you should believe in The Book" (Aigner and Ziegler 2001, Preface). If in the kitchen of mathematics people use religious language, does it mean the two fields are related?

Probably the most famous example of direct use of the term “theology” in relation to mathematics can be found in the reaction of Paul Gordan, in 1888, to Hilbert’s non-constructive proof of the theorem on the existence of finite bases in some spaces. Gordan said, „Das ist nicht Mathematik. Das ist Theologie.” Later, having witnessed further accomplishments of Hilbert, he would admit that even “theology” can be useful (Reid 1996, p. 34, 37).

One should not miss the expressions that seem to point to an overlap of the contents of mathematics and theology or at least of the presence of common features. This possibility is mentioned in some official statements, made for the “front” public. When Hilbert mentioned “the paradise” he could have meant only a rhetorical effect, but in another classic exposition of “Hilbert’s Program,” in 1930, Rudolf Carnap, while talking about logicism, used the phrase “theological mathematics.” According to him, Ramsey’s assumption of the existence of the totality of all properties should be called theological mathematics in contradistinction to “anthropological mathematics” of intuitionists; in the latter, all operations, definitions, and demonstrations must be finite. Ramsey “elevates himself above the actually knowable and definable and in certain respects reasons from the standpoint of an infinite mind which is not bound by the wretched necessity of building every structure step by step” (Benacerraf and Putnam 1983, p. 50). Clearly, when infinity is considered, theology is most easily invoked. Or, at any rate, this was happening in the late nineteenth and the early twentieth centuries. Back then it was still far from obvious that we could act as if we could fully control infinite structures, perform infinite sequences of operations, and consider as given the results of an infinite number of acts. By now, such capacities are commonly seen as unproblematic. On a mathematical level, the opposition to infinitistic mathematics seems virtually non-understandable for a contemporary university student. For example, the rejection of the Axiom of Choice by French “semi-intuitionists” was explained, in 1908, by Emile Borel: “the legitimacy of a non-denumerable infinity of choices (successive or simultaneous) … appears to me … entirely meaningless” (after Moore 1982, p. 102). Nowadays, only the rare species of committed constructivists can express such objections.

The present situation resulted from an exceptional, total victory of Cantor’s ideas. Infinitistic mathematics is now hardly seen as connected to theology. Almost always, the theological connection is completely ignored. It is only deep at “the back” that we could say that only God knows the entire decimal representation of the number π. If we were to say that “at the front,” we would stress it is just a joke, as did Erdös.

3 Historical Interrelations

Through the eighteenth century nearly everyone was religious, so no wonder mathematicians also were; and later there existed professional mathematicians who were deeply religious; this is still the case. It seems, however, that for many of them the two dimensions of their lives are not genuinely connected but rather just present side by side. The point is whether for some of them the two components could be in harmony. What this can possibly mean in general is hard to say. It is possible, however, to present telling examples.

3.1 Pythagoreans and Their Legacy

The best known and most highly consequential ancient case is that of the Pythagoreans whose founder was both a mathematician and a religious leader. Around 530 B.C.E., Pythagoras began to attract disciples in southern Italy. The Pythagoreans saw (natural and rational) numbers as the essence of everything – things, relations, musical harmonies, and the celestial harmony, produced by the spheres containing heavenly bodies; at the same time they discovered – to their alarm – incommensurables, or realized that there existed irrational numbers. (For details see Fritz 1945.) The most directly religious aspect of their views can be seen in their veneration of the tetractys, or the number 10 represented by the points arranged in an equilateral triangle formed by rows of 1, 2, 3, and 4 points. The upper point represented the essence of all numbers, the two below symbolized the line, the next three symbolized the two-dimensional triangle, and the last four stood for a pyramid. The tetractys was seen as revealing the structure of reality. It was considered holy, and the Pythagoreans swore by it. (Fritz 1981, p. 221.) One of the prayers was, “Bless us, divine number, thou who generates gods and men! O holy, holy tetraktys, thou that containest the root and source of eternally flowing creation!” (Dantzig 1954, p. 42).

The Pythagorean view of the role of numbers sounds far too mystical for present-day scientists and philosophers, yet not only did it remain extremely influential throughout centuries but it has reemerged in a new format in our own time. A century ago, Bertrand Russell wrote that “perhaps the oddest thing about modern science is its return to Pythagoreanism” (Russell 1924, p. 252). He meant, for example, quantum physics allowing only a finite number of states, which brings back the idea that everything is composed of definite units. This idea is further developed in modern physics. For example, Max Tegmark (2014) claims that reality is a mathematical structure. Russell also referred to modern genetics and to the arithmetization of the calculus, leaving only natural numbers as the basis of the continuous. For us, the arithmetization of syntax and especially the ubiquitous presence within our world of the digital realm, which is fundamentally a configuration of numbers, has opened up a new reality that deserves the name of “modern Neopythagoreanism” (Krajewski 2011). Its climax can be seen in the thesis presented by Stephen Wolfram, the author of the widely used program Mathematica. In his 2002 book *A New Kind of Science*, he claims that linear automata can code the most complex calculations including the universal Turing machine. (For a critique, for instance of the issue of the complexity of de-coding, see Baldwin 2004.) This claim makes possible the vision of the universe being a computer or even a computer program. Thus, as had been claimed by Pythagoras, literally everything would consist of numbers. To be sure, most of us see no directly religious aspect in such pancomputationalism. Yet for the ancients the interplay of the religious and the mathematical was obvious.

Plato extended Pythagorean ideas, ascribing to mathematical concepts a special role. (See, for example, Baum 1973.) Plato’s philosophy has been at the center of Western thought and mathematics plays an important role in his thought as an example of certain knowledge and also as an educational tool required in his Academy, established at the beginning of the fifth century B.C.E., the first institution of higher learning. According to (later) tradition, the famous inscription at the entrance to the Academy read: “Let no one ignorant of geometry enter.” Mathematical examples are presented in Meno to prove both the existence of immaterial Forms, which is a central point in Plato’s philosophy, and the preexistence of the soul, a religious claim. In the late dialog Timaeus the divine “demiurge” is presented as using mathematics to form the material world. And it is mathematical entities that are in between the ideal Forms in the "divine" realm and the visible objects in our material world. Mueller says, "Plato did have a comprehensive picture of the cosmos divided into a higher divine, intelligible world and a lower human, sensible world … A, and I suspect the, crucial link between these worlds is mathematics" (Mueller 2005, p. 117).

Theological presentation of the Platonic vision became common. In pre-Christian Rome, Seneca wrote that numbers existed in the divine mind: "God has these models of all things within himself, and has embraced the numbers and measure of all things which are to be accomplished in his mind. He is filled with those shapes which Plato calls 'Ideas': immortal, immutable, indefatigable" (after Albertson 2014, p. 56).

Around 100 C.E. Nicomachus of Gerasa wrote Introduction to Arithmetic, soon translated into Latin. "Nicomachus's works represents the fruition of Pythagoreanizing Platonism in Greek antiquity and the realization of what we can call a mathematical theology" (Albertson 2014, p. 51). It became highly influential: "the continued use throughout Europe of the arithmetic of Nicomachus, as translated by Boethius, are typical of the period from the tenth to the sixteenth century," state Robbins and Karpinski (Nicomachus 1926, p. 145). According to them, "the philosophical arithmetic of the Greeks, of which the arithmetic of Nicomachus is a specimen" includes number theory as we understand it now and at the same time "we have a mystical development, ascribing even magical powers and life properties to numbers" (Nicomachus 1926, p. 4). Indeed, such numerology was present in many cultures as was "sacred geometry," especially insights used in the design and building of religious constructions. In the ancient world also gematria was used, ascribing numbers to words, which is possible due to the fixed numerical values of letters. It was applied primarily to religious texts and is still mentioned in traditional Judaism (Cf. Grattan-Guinness 2009, Chap. 14).

3.2 Medieval Developments

Another Pythagorean achievement, not appreciated today, reigned supreme for centuries. It is the classification of mathematics into the discrete and the continuous with the former involving arithmetic and music, and the latter geometry and astronomy. This was the basis for the famous quadrivium of knowledge, which structured education, religious and nonreligious, in Europe until the early modern era. Its religious significance is stressed by Albertson: "the quadrivium as originally conceived for Latin Christianity by Boethius named the universal principles of first philosophy, or fundamental theology" (Albertson 2014, p. 10). In the fifth century C.E., in the prologue to his commentary on Euclid's Elements, Proclus explained that "mathematicals" are "projections of higher principles within soul, just as soul itself is a projection of the divine Mind." Hence mathematics is secondary to theology, and "the highest bond of the quadrivial arts is the divine Mind itself" (Albertson 2014, p. 66). Proclus also believed that "mathematics is the best conceptual language for expressing the ineffable" (Albertson 2014, p. 66). The connection of theology to mathematics was very strong.

And it remained so for many centuries, although not for everyone. In the fourteenth century, Nicole Oresme, who was a scientist, philosopher, and theologian, presented the first proof of the divergence of the harmonic series. His mathematical work was separate from theology, unlike the scholarly work of Nicolas of Cusa in the fifteenth century. "Cusanus viewed his theological and mathematical explorations as belonging to the same integrated intellectual enterprise." He was praised in surprisingly strong words: "how is even Aristotle comparable to this Cusanus?" wrote Giordano Bruno, and added, "I would readily acknowledge his mind to be not just the equal of Pythagoras, but far superior" (Albertson 2014, p. 1).

Nicolas continued the earlier idea that the proofs of arithmetic, music, geometry, and astronomy lead to knowledge of the Creator. In particular, "mathematicals provide the surest pathway to contemplating God" (Albertson 2014, p. 171). For example, his book De docta ignorantia, or "On Learned Ignorance," of 1440, contains a presentation of geometrical figures as tools helping in contemplation, based on the idea that "mathematical knowledge is a via negativa to God" (Albertson 2014, p. 181). Thus, given a sequence of circles tangent to a given line in the same point, when their radii grow indefinitely the limit is a line, the common asymptote. And similarly with other figures, for instance indefinitely growing triangles. Therefore, "if there were an infinite line, it would be a straight line, a circle, and a sphere" (Nicholas of Cusa 1985, p. 20). In another work, he describes the game of a spinning top. "If the top spins with 'infinite velocity,' then any two points on the circumference of the top's surface would be co-present with any given point on the fixed circle" (Albertson 2014, p. 259). All the examples show that infinity is not present in our world, and cannot be reached from the domain of the finite. Even potential infinity is impossible. Yet it can be grasped conceptually. The reason for this is the presence of infinity in God. It involves coincidentia oppositorum. And since infinity is never reached via approximations by finite objects, it is infinity that is primary: "the one and only Absolute Eternity is Infinity itself, which is without beginning. Consequently, everything finite is originated from the Infinite Beginning" (Nicholas of Cusa 1996, p. 538). And mathematics can reflect God's mind: "we give our name 'number' to number from the Divine Mind" (Nicholas of Cusa 1996, p. 552).

All such considerations strike us now as rather naïve mathematically. Perhaps the most appealing image, also discussed by Cusanus but present since at least the twelfth century, and perhaps much earlier, is offered by the concept of the infinite sphere, whose center is everywhere and whose circumference nowhere. This Latin statement "Deus est sphaera infinita cuius centrum est ubique, circumferentia nusquam" (Liber XXIV philosophorum II 1989) can evoke the concept of the circle at infinity in the projective plane. (See Sect. 5 below for a contemporary insight.) The picture can have far-reaching consequences. According to Counet, "Nicholas's universe, which is actually neither infinite nor finite, whose centre is at once everywhere and nowhere, which is not exactly describable through mathematical notions, although mathematics can give an ever more precise account of it … the revolution initiated by Nicholas of Cusa is far more radical than that of Copernicus" (Counet 2005, p. 284–5).

This infinite circle was meant as one of the definitions of God. Cusanus also tried to present various mathematically conceived names of God, for example, "infinite Angle" and "infinite Number." He claimed that "an infinite angle would enfold all opposing angles, maxima and minima" (Albertson 2014, p. 247).

3.3 (Early) Modernity

Galileo's famous dictum that the Book of Nature is "written in the language of mathematics" did show the scientific way independent of religion. It is common to treat Galileo as the initiator of modern secular science who was silenced for his anti-Church position. Yet he remained a believer and did not think his views were heretical. In his letter of 1615, he wrote that "the true meaning of the sacred texts … will undoubtedly agree with those physical conclusions of which we are already certain and sure enough through clear observations or necessary demonstrations" (after Remmert 2005, p. 353). What is more, Galileo wanted to convince everyone that the mathematical approach is necessary and, according to Remmert, he referred to "the metaphysical legitimation he constructed for the mathematical sciences by eventually taking recourse to God and the divine" (Remmert 2005, p. 349). Similarly, Johannes Kepler "even considered the mathematical way as a kind of divine service" (Remmert 2005, p. 356). And the celebrated founder of mathematical physics, Isaac Newton, was not only deeply involved in theological research, but he should be seen as someone working not in modern science but still in the field of natural philosophy that "considers God and His creation. This orientation towards God was taken for granted" (Pater 2005, p. 463).

Another giant of that era, Leibniz, seems to indicate a much more specific connection of mathematics and theology. His dictum, "As God calculates and executes thought, the world comes into being" (after Breger 2005, p. 489), presents God as a mathematician. Again, he employs theological language to indicate the role of mathematics in the scientific account of the world. Leibniz believed that the same mathematical approach can be applied in theology. By that, he meant the need to use precise logical arguments rather than mathematics as such. One can still ask whether Leibniz applies mathematics in his theological considerations, that is, if it is possible to connect Leibniz's mathematical achievements, such as the creation of contemporary infinitesimal calculus, with his theodicy. An attempt to establish a link has been made. Namely, the idea of our world being the best of possible worlds is an expression of the belief in the applicability of an optimization principle. According to Emil du Bois-Reymond, Leibniz "conceives God in the creation of the world like a mathematician who is solving a minimum problem, or rather, in our modern phraseology, a problem in the calculus of variations – the question being to determine among an infinite number of possible worlds, that for which the sum of necessary evil is a minimum" (du Bois-Reymond 1870, p. 36). Max Planck liked this interpretation, and actually, this approach can be generalized since it is known that many physical theories can be seen as applications of the principle of the least action. Leibniz can be perceived as a forerunner of this approach not just with regard to physical theories but also in the realm of theology. Yet, one can ask, how essential is mathematics here? The mathematical terminology does not seem to add anything essential to the general idea that our world is the best of all possible worlds. To express this idea no mathematics is needed. Thus, while it is possible that Leibniz was motivated by ideas similar to his mathematical concepts, the connection appears to be primarily psychological. Possibly important in the context of discovery, the connection plays no role in the context of justification.

Similarly, Blaise Pascal who made great contributions to mathematics devoted a lot of attention to religious matters. He emphasized the opposition and the lack of continuity between these areas: the reasons of the mind are opposed to the reasons of the heart, esprit de géometrie is opposed to esprit de finesse, and, even more important, he knew since his mystical experience of 1654, that the God of Abraham, Isaac, and Jacob is to be distinguished from the God of philosophers. A mathematical mind and the experience of a man who created the calculus of probabilities can be seen in his famous theological argument, the wager, le paris de Pascal. While some mathematical inspiration might have been important for Pascal in his theological considerations, this was a general, nonspecific encouragement. The formulation of it does not require mathematics, even if strict mathematical formulation can be helpful. Actually, "probability theory does not seem to have been invoked to elaborate" the high probability of the existence of a Designer, notes Grattan-Guinness (2009, p. 305).

George Boole was a founder of modern symbolic logic that became the foundation for the epitome of our era, computers and the digital realm. Yet, again, writes Daniel Cohen, "it is wrong to assume that the purpose of nineteenth-century pure mathematics and the symbolic logic that arose out of it was to construct a completely scientific, secular realm of philosophy" (Cohen 2007, p. 11). According to his wife, for Boole mathematics and logic was of secondary interest; primary was "the cause of what he deemed pure religion" (Cohen 2007, p. 77). He "seemed never tired of reading what are called 'religious books.' … The pursuit of scientific truth was also a religious act" (Cohen 2007, p. 92–93).

Another key logician of that time Augustus De Morgan shared much of Boole's religious approach, yet in the late nineteenth century he "provided a secular approach to those issues that would become increasingly alluring to the growing ranks of professional mathematicians. In a charged cultural environment, De Morgan realized it would be best for everyone if religion and mathematics did not mix" (Cohen 2007, p. 136). In England, Russell completed the divorce, and in France, it was the influence of the Enlightenment that had a strong anti-religious impact. Yet for the creators of the Encyclopédie both theology and mathematics were seen as marginal branches of the tree of knowledge. Perhaps the reason was that "both mathematics and theology moved beyond the reach of experience into realms of the perfect and the infinite" (Richards 1992, p. 54).

Some mathematicians are also theologians. Some theologians use mathematical concepts or expressions with mathematical connotations. How deep is the connection of theological discourse with mathematics? Often it seems superficial and inessential, but sporadically the suspicion can arise that a serious relation of contents or method can be detected.

Pascal, Newton, Leibniz, and Boole were leading mathematicians and scientists and at the same time men of faith who, in addition, were devoting a lot of efforts to theology. They all thought that the search for mathematical truths makes it possible to uncover God's thoughts. However, one can doubt if they revealed a genuinely essential connection between the two fields. Can we find a stronger connection between mathematics and religion? In the next Section some examples of possible religious inspirations are presented, and in Sect. 5 mathematical models in theology. Before this, the most important case of an interplay of religion and modern mathematics is presented, that of Cantor.

3.4 Cantor and Infinity

Infinity, understood as a completed entity, was for a long time seen as representing the nature of God. It was basically a theological concept. Mathematicians used potentially infinite processes: extending lines, adding numbers, dividing geometrical objects. The proposals that actual infinities could be studied by mathematicians were not accepted. In the early nineteenth century Gauss, the prince of mathematicians, rejected infinite collections. And then came Georg Cantor. At first, he was a regular mathematician. Studying the sets of convergence of trigonometric series, he introduced the notion of a derivative of a set – that is, the collection of the limit points of sequences with members belonging to the given set – and he noticed that we can consider the intersection of the infinite sequence of subsequent derivatives. What is more, one can take the derivative of the intersection, then the derivative of the derivative, etc.

This led him to the notion of a transfinite sequence and transfinite ordinal number. Soon, a whole theory of infinite sets was developed by him and later by others. For several decades, however, mathematicians were against the new theory. The strongest criticism was voiced by the highly influential Leopold Kronecker who was defending the traditional finitism, that is, the view that only finite objects really exist. Cantor claimed that his infinite numbers are of the same nature as the irrational numbers, since both kinds of numbers are the limits of infinite sequences. And both can be accepted because they satisfy certain conditions, which, according to Joseph Dauben, should be interpreted as consistency (Dauben 1977, p. 88). In addition, Cantor had deeper reasons to believe in the reality of his infinite numbers. Deeper means theological. In an 1895 letter to Hermite, Cantor referred to St. Augustine stating that natural numbers "exist at the highest level of reality as eternal ideas in the Divine Intellect" (Dauben 1977, p. 94; also Dauben 1990). Having adopted this view, it is easy to accept the existence of arbitrary infinite objects; after all, there is no reason to impose any limitations on the "divine intellect."

Cantor faced a wall: his approach was rejected not only by Kronecker or by Charles Dodgson, better known as Lewis Caroll, the author of Alice in Wonderland, who derided the "bewildering region of Infinities and Infinitesimals" (Cohen 2007, p. 174); even one of Cantor's allies, who had popularized his mathematical results, Magnus Mittag-Leffler advised him, in 1885, not to publish a paper on order types. As a result, Cantor was pondering withdrawal, abandonment of mathematics. What he did instead was to start an intensive correspondence with theologians. He wrote to Mittag-Leffler that his ideas about infinity were inspired by God. He also stated that by his work he wanted to serve the Catholic Church. His approach was met with interest by some neo-Thomist theologians. For example, Constantin Gutberlet used Cantor's set to argue that actual infinity exists. In correspondence with him, Cantor argued that his infinite numbers make God greater, not smaller. Cantor was able to gain the acceptance of Cardinal Johann Franzelin, a leading theologian of the First Vatican Council, who was satisfied by the Cantorian distinction between absolute infinity and actual infinity. According to Cantor, some infinite structures are so infinite that they are not sets, that is, they cannot be subjected to set-theoretic operations. Due to this distinction, antinomies could be avoided. For instance, the ordinal number ω and the set of natural numbers are actually infinite, but there is no set of all ordinal numbers, because as a "multiplicity" it is "absolutely" infinite. Franzelin accepted the idea that the Absolute Infinite is reserved for God and the (mere) actual infinity is present in the creation. He wrote to Cantor: "there is no danger to religious truths in your concept of the Transfinitum" (Cantor 1932, p. 386).

Cantor firmly believed in the existence of infinite sets. At the same time, he categorically rejected infinitesimals, the infinitely small quantities. He alleged that "the atoms of our physical world form an actually infinite set with cardinality $\aleph 0$ and he believed that the set of the atoms of the universal ether is essentially bigger and possesses the power $\aleph 1$ of the continuum" (Thiele 2005, p. 535). (He assumed the continuum hypothesis.) He was not only deeply interested in theology – for example, he wrote a pamphlet rejecting the immaculate conception – but also was convinced that he was "merely the tool of a higher power" (Thiele 2005, p. 546).

It is apparent that Cantor not only used theological arguments to support his theory, but he also found consolation among Catholic theologians. Granting this, one can still ask whether theologians and mathematicians speak about the same realities. This is unclear (See Oppy

2011, Tapp 2011). Did Cantor use the theological notions in his mathematical considerations? It is doubtful that his religious considerations were necessary or even essential.

The Cantorian distinction between the "regular" and the "absolute" infinity was mathematically right. It gave rise to the mathematical distinction between sets and proper classes, or collections that may not be elements of other classes. However, Cantor's theological-philosophical justification of the distinction did not stay. For him, any set, finite or infinite, could be treated as unity (Einheit), or one completed whole, whereas those absolutely infinite collections could not be treated as completed unities. Yet von Neumann, Bernays, Gödel formulated axiomatic theories of classes in which both sets and proper classes, like the totality of all sets, are treated as objects in the same way; they all can equally be treated as unities.

The story does not end here. One can imagine the Cantorian-style procedure continuing. Christopher Menzel invokes this possibility as a comment upon one proof of the existence of God proposed by Alvin Plantinga 2007. The proof takes for granted a later development of the Cantorian vision, that is, the generally accepted cumulative hierarchy of sets: they are formed iteratively by collecting sets, and possibly other objects, formed at previous stages; the sequence of stages is transfinite. Incidentally, this hierarchy uncovers the role of theological categories: the void and infinite power. Namely, from the empty set one can create "everything," or rather the universe of "pure" sets sufficient for the foundations of mathematics. Nothing, emptiness, is combined with infinite power and a kind of unrestricted will to continue the construction ad infinitum. Terms like "infinite power," "all-powerful will" are unmistakably theological. If Leibniz had known modern set theory, he would have rejoiced, both as a theologian and as a mathematician. He claimed that "all creatures derive from God and nothing" (Breger 2005, p. 491). When he introduced the binary notation, he gave theological significance to zero and one: "It is true that as the empty voids and the dismal wilderness belong to zero, so the spirit of God and His light belong to the all-powerful One" (after Barrow 2000, p. 33).

Now, if idealized "collecting" is treated as executed by an infinite mind, the process can be extended: all sets form the universal class and beginning with it one could consider classes of classes, and form a superhierarchy, and then to continue it further into the transfinite. As Menzel notes, for the realist who believes in the existence of sets (as most mathematicians seem to do) it is unclear "why the cumulative hierarchy is necessarily only as "high" as it is in fact and could not be extended. Call this the realist's impasse" (Menzel 2018, p. 44). His solution would surprise mathematicians: "a satisfying explanation for the existence of the sets in any given world: they are the ones whose members God in fact chose to collect. … given that God knows all of the large cardinal properties – certainly at least those we are in principle capable of formulating – it is reasonable to suppose that God does enough collecting to satisfy all of them" (Menzel 2018, p. 49–50). This discussion shows a lasting religious potential in the Cantorian vision.

4 Religious Sources of Mathematical Concepts

There are examples of mathematical concepts that according to some authors have been developed due to religion. None of these instances gives an undoubted exemplification of fundamentally religious origins. Yet each of the cases mentioned below – Indian origins of mathematics and zero, Christian sources of the initiation of the algebraic approach, Maxwell's stress on relations, Grassman's theological inspirations, the creative function of naming – does provide a vista on something religious.

Basic religious behavior involves ritual. Often such rituals are ancient and their sources are of mythic character. Abraham Seidenberg in (1961, 1962), and later papers (e.g., 1978) presented the theory that religious rituals were the source of mathematics. The need to perform ceremonies, for instance, those presenting myths of the origin, led to both the art of calculation and basic geometric notions. Independently of this idea, Seidenberg advocated the diffusion theory: fundamental mathematical concepts were introduced once and gradually migrated to other places to eventually reach the whole globe. In this way, he opposed those historians who believe that basic notions were discovered independently in several places. The diffusion claim was approved by Bartel van der Waerden: "if we find quite similar ideas about the ritual importance of geometrical constructions in Greece and India, … the conclusion that these religious and mathematical ideas have a common Indo-European origin is highly probable" (Van der Waerden 1983, p. 35).

In addition, Seidenberg claims that it was ancient India that was the hypothetical source of basic arithmetic and geometric concepts. This is claimed with reference to interpretations of some old Vedic writings. To give one illustration, "most mathematical problems considered in the Śulba Sūtras spring from a single ritual requirement; namely that of constructing altars that have different shapes but occupy the same area" (Rogers 2015, p. 272). From India, the mathematical knowledge was brought to Babylonia. Seidenberg, a distinguished algebraist, was a major scholar so his proposals should be treated with attention, even though they met with serious criticism. Yet, more recently, further research of ancient mathematics has shown that "some of Seidenberg's opinions may have been justified," says Leo Rogers (2015, p. 269).

While ritual applications could stimulate research, it is less than clear that they were the source of mathematical ideas. A great example of the religious source of concepts, and an Indian one at that, is provided by the invention of zero. Zero as a full-fledged number – a more substantial idea than just an indication of absence in notation systems – emerged in India, not later than fifth century CE. In 628 Brahmagupta presented the rules of using zero in addition and multiplication. And from there it migrated to the Islamic centers and to Europe. The term "zero" was derived from the Arabic assifr, meaning void, the translation of Hindu sunya (See Barrow 2000).

It is impossible to overstate the importance of zero. It is necessary for the full treatment of arithmetic, for the mature functioning of the decimal notation of numbers, for the operations like binary arithmetic that made computers conceivable. According to John Barrow, "the Indian system of counting is probably the most successful intellectual innovation ever devised by human beings" (Barrow 2000, p. 69). It is so essential and seems to us so simple. Why did the introduction of the number zero take place in India and the brilliant Greek mathematicians not have the idea? One might say that it just so happened, for no special reason. It is much more illuminating to detect cultural circumstances that enabled the innovation, more specifically, the features of Indian culture that were absent in Greece, and helped develop the notion of zero. The relevant difference can be seen in the treatment of nothingness. Zero symbolizes nothing. For Greek philosophers, nothing can result from nothingness, one cannot reasonably state about non-being anything more than its nonexistence. Against that logic, the Indian "religions accepted the concept of non-being on an equal footing with that of being. Like many other Eastern religions, the Indian culture regarded Nothing as a state from which one might have come and to which one might return – indeed these transitions might occur many times, without beginning and without end" (Barrow 2000, p. 65). To have the concept of zero one needs to treat nothing as something, absence and presence at the same time. The Hindu idea of a "positive nothing" indicates a complex notion involving various phenomena, from an empty container to a mystical void. This concept is expressed by the term sunya. It is a consistent idea, and despite its contradictory description as non-being and being at the same time, it can be represented by a symbol, zero.

Barrow's account is echoed by William Byers (2007) who treats it as an instance of a more general rule. He argues that some contradictions play a constructive role in mathematics. In particular, new concepts are introduced to overcome incompatible frames of reference. He states, "our culture has tamed the concept of zero by removing its contradictory aspects and incorporating it into a logically coherent mathematical system." Of course, he does not deny the need to have consistent theories. Yet he adds, "reality in general and mathematics in particular are greater than logical consistency" (Byers 2007, p. 109.)

The above account of the history of zero seems to be the best available. The role of religion, Hinduism, has turned out to be essential and non-trivial. A conscious effort of discovering vital Christian sources was undertaken by Gene Chase. He proposed a few examples "wherein Christian theology potentially provided specific insights and motivations for the development of specific new branches of mathematics" (Chase 1996, p. 193). One of them is Cantor, another Boole, both treated as unproblematic cases. Two other stories are of special interest since specifically Christian motives are invoked.

First, William of Ockham (fourteenth century) "wanted to be faithful to the doctrine of transubstantiation but to be consistent with observation. He argued, therefore, for a philosophy which allows the substance to change from bread to body, while the attributes (accidental properties) do not change" (Chase 1996, p. 198). And "his views on the Eucharist were a theological catalyst for his algebra" (Chase 1996, p. 197). Ockhamm "freed mathematics from being about specific concrete objects, or even about specific objects in Plato's heaven" (Chase 1996, p. 198). This is the initial source, claims Chase, of our contemporary mathematics dealing with abstractions, independent of substance.

The second example is provided by the bold move made by James Clerk Maxwell who introduced the concept of field. In the 1860s, he wrote "that an electromagnetic field acts through all space and all time, not just where matter is present. Lord Kelvin called Maxwell's view 'mysticism,' for in it even time and space were contingent upon God's continual sustaining" (Chase 1996, p. 206). Today we think of physical fields as material. Maxwell's view was understood as being non-materialistic, because it was against the mechanist reading of materialism. It was possible for him to make that move, claims Chase, because "in the Calvinistic tradition, God is the author not only of the material universe but also of the laws governing that universe. … [Maxwell's approach] has completely reversed the roles of things and their relations" (Chase 1996, p. 206).

In mid-nineteenth century, another development took place, important for later mathematics and related to theology, as has been recently pointed out (Lewis 2011; Achtner 2016). Hermann Grassman in Ausdehnungslehre of 1844 introduced revolutionary concepts that, albeit not immediately, initiated vector and tensor calculus and modern linear algebra. “Ernst Cassirer contended, … that it was Grassmann who changed the understanding of modern mathematics as a science of magnitudes to a science of structures, functions or relations” (Achtner 2016, p. 112). The point is that Grassmann was primarily a theologian. What is more, he credited the possibility of introducing his new concepts to his teacher Friedrich Schleiermacher, a leading German theologian of that time, who taught the right approach, and provided his students with the appropriate method. This sounds like a confirmation of the theological source of multidimensional structures, but unfortunately, Grassmann’s specific explanations are very general. According to Achtner’s summary, “New knowledge is found by constructive abstraction from diversity to the complexity of unity going through different levels of abstraction leaving behind perturbations at a particular level” (Achtner 2016, p. 117). The theological aspects of such statements are unclear, and a scholarly debate continues with regard to the claim that Schleiermacher had a genuine influence on Grassmann.

Two more examples come from the beginning of the twentieth century. Intuitionistic mathematics, created by Luitzen E. J. Brouwer, denies non-constructive entities and even the logical law of excluded middle. At the origin of this vision, there were mystical considerations, important for Brouwer himself. His doctoral dissertation was purged of those thoughts since his advisor did not count them as mathematics. Almost all mathematicians would agree. Yet, the most authoritative study of Brouwer, van Dalen (1999), is entitled “Mystic, Geometer, and Intuitionist,” in this order. The story is even more ambiguous. Brouwer was pressed by his mysticism to abandon mathematics as “evil,” but his attraction was too strong. In the words of his biographer, “the genesis of the dissertation is a story of temptation, as experienced by all hermits and saints. No matter how much Brouwer fought the evil influence of the world, the fascination of mathematics proved stronger than his Spartan views” (van Dalen 1999, p. 84). It can be said that his research was conducted despite his religious views rather than because of them. At the same time, his mystical predilections could play a role in devising mathematical constructions: “Brouwer’s view of the continuum shows a suggestive similarity to the mystic experience of the initial chaotic state of the individual. In this state, also, there are no sharp bounds, everything is flowing and amorphous” (van Dalen 1999, p. 114).

Sources of inspiration should not be ignored, but if they are religious, it is indeed unclear how to account for them, be it in the case of Grassmann or Brouwer. An especially clear case, although much less familiar, is that of Russian mathematicians who about one hundred years ago were developing the so-called descriptive set theory. Some of the scholars of that group, in particular its founder, Nicolas Luzin, were inspired by a heretical (for the Christian Orthodox Church) theology Imiaslavie (name worshipping). The story is now known due to the book Graham and Kantor 2009. The theology of name worshipping was heretical because it involved the readiness to identify God with God’s name. Luzin, later the head of the Moscow school of mathematics, was influenced by theologian Pavel Florensky who noticed that making God present by an appropriate uttering of God’s name is similar to Cantor’s claim that a proper naming of an infinite set is sufficient to ensure the set’s existence. At the beginning of the twentieth century, it was highly problematic to claim that the naming of an infinite ordinal number is enough, no other proof of its existence is needed. Now, this claim is widely accepted. One of the ways to its acceptance was Luzin's conviction that university stress on logical and empirical sources of knowledge should be supplemented with "intuitive-mystical understanding" (Graham 2011, p. 161). This religious approach made possible the development of a major mathematical school. While also in this case one can claim that the same research could have developed without that mystical background, the fact is that a hundred years ago some religious ideas and practices encouraged new mathematical constructions.

A more recent parallel has been proposed: Alexandre Grothendieck, the creator of a massive framework that has deeply influenced modern mathematics, was also using the method of creative naming and, at the same time, spoke of a mystical vision of the mathematical universe. "Grothendieck, like Luzin, placed a heavy emphasis on 'naming,' seeing it as a way to grasp objects even before they have been understood" (Graham and Kantor 2009, 200).

Finally, a famous case must be mentioned, that of Indian genius Srinivasa Ramanujan who, a hundred years ago, produced unbelievably complicated theorems, claiming that they were revealed to him by goddess Namagiri. She "would write the equations on his tongue. Namagiri would bestow mathematical insights in his dreams" (Kanigel 1992, p. 36). Ramanujan's story is so exceptional that it is impossible to generalize it.

5 Mathematical Models in Theology

Mathematical illustrations of theological concepts are possible. Those of Cusanus or Leibniz (see Sect. 3) look rather naïve from the present-day perspective. The same can be said about explaining the Trinity by $\aleph_0 + \aleph_0 + \aleph_0 = \aleph_0$, where the symbol "aleph-zero" stands for the first infinite cardinality. (After all, $\aleph_0 + \aleph_0 + \aleph_0 + \aleph_0 = \aleph_0$ also holds, which means that the concept of a "Quadrity" would be equally well justified.) Moreover, it is doubtful that it makes sense to refer to mathematical addition of numbers, finite or infinite, as corresponding to the "addition" of the persons of the Trinity. Perhaps it would be better to refer to the operation of the Boolean union: in Boolean algebras adding the (Boolean) one to itself results in the same one. In addition, it is the only element (of the algebra) that is greater than each of the remaining ones. It has, therefore, a "divine" quality, and since it is unique, it can be seen as modeling monotheism. Yet, obviously, this model is as shallow as it is arbitrary. It is no better than Cusanus's reverence for "the equation $1 \times 1 = 1$ as a sublime name of God" (Albertson 2014, p. 1). Nowadays, mathematical models are so rare that in the monumental volume Diller and Kasher 2013, presenting dozens of models of divinity, they do not appear at all. Yet, mathematical models are sometimes invoked by modern theologians. A notable example appeared in the Victorian era: Flatland (Abbott 1952) is a tale about geometric creatures living in a flat world. A three-dimensional being has divine properties from the flatlanders' perspective: it can perceive the whole flat world and when it interferes with the plane this is perceived by the flat figures as a miraculous happening. In this way, the recently discovered multidimensional geometry is presented as not a threat but rather an aid to religious faith. The words of Boethius (sixth century) become understandable: "God abides for ever in an eternal present, … and, embracing the whole infinite sweep of the past and of the future, contemplates all that falls within its simple cognition as if it were now taking place" (Boethius 1897, Book V, Song VI).

To give a contemporary example, James Miller 1998 presented a model, based on the normal (Gaussian) distribution, in order to explain the supposedly paradoxical belief that submission to God's will provides more freedom than does disobedience. While this is difficult to explain in everyday language, it becomes understandable when the Gaussian curve is considered in the space of all options available at a given moment of one's life. On the y-axis the number of options is indicated and on the x-axis their quality. Miller assumes that the most agreeable to God are the options belonging to the "middle way." The acceptable options are not much different from those in the middle, distant no more than one standard deviation. As a result, we model free will, since there are many possible options to choose from. At the same time, since the columns near the middle position are relatively larger, the way that pleases God offers more opportunities. The submission provides more freedom. Furthermore, sin can be illustrated as a deformation and dislocation of the Gaussian curve: then what the sinner takes to be the middle way is in reality distant from the genuine "divine" middle way.

The value of the model is debatable. It is commendable if it helps someone to grasp some theological ideas.

Another proposal has been made, in a way opposite to the well-known idea that the set of true sentences is part of the "divine" knowledge. Making use of logical and set-theoretical notions one can try to express "divinity" as … the set of all true sentences. John Post (1974) does it in the spirit of Quine's minimalism in order to express the foundations of theism, avoiding "metaphorical, analogical, symbolic, mythical" (Post 1974, p. 736–7) talk about God. He tries to paraphrase the statements about God in a possibly restricted language so that one can get sentences that are "literally true." Thus "Godhead" is defined as the smallest set G, whose transitive closure (that is, its elements, elements' elements, etc.) includes the set of all true sentences about sets, the set of true sentences about justice, virtues, religious experience (whatever it may mean for us), etc. The set G is mysterious, it "combines transcendence with immanence" (Post 1974, p. 740), it contains complete knowledge, and the "mythic" personal God is its "intended object." Even this short summary suggests that this construction can be easily criticized as contributing nothing essential to theology. The model can be easily undermined by asking, e.g., whether devout people really pray to a set.

Another model is presented in Krajewski (2019) (and mentioned in 2011). It illustrates Martin Buber's vision of God who is revealed in interhuman relations. In 1923, he wrote, "The extended lines of relations meet in the eternal Thou" (Buber 1937, p. 75). To anyone acquainted with projective geometry, this picture would seem familiar. In the system of plane geometry, the direction of each straight line constitutes a point of a new type, and this point at infinity is common to all parallel lines. Thus, every pair of straight lines (in the plane) intersects in exactly one point, with no exceptions. For parallel lines this point of intersection is at infinity, but in projective geometry, these infinitely far located points have the same status as do the usual points of the plane. What is more, the totality of the points at infinity constitutes one "straight line at infinity." This line can be seen to correspond to God in Buber's vision: the line of the relation between two arbitrary individuals always reaches the same God.

The use of this picture can be extended. The well-known model of the projective plane in the Euclidean (three-dimensional) geometry is given by a hemisphere. If the Northern hemisphere together with the equator is imagined all the points above the equator constitute the proper (finite) points of our model, and the points on the equator constitute the points at infinity. Straight lines are the semicircles with the center coinciding with the center of the ball (the Earth). (They form the shortest way between two points, and are called great circles.) One more move must be made. In order to have only one point corresponding to the direction of a line, the opposite points of the equator must be identified. The resulting objects cannot be easily imagined, but it is also possible to assume that the whole line segment joining two opposite points constitutes, *ex definitione*, one "improper point." The totality of these improper points forms a plane figure, or a disc determined by the equator. In our model, this object is the basis of the hemisphere. Thus, God is here the fundament of our world, unattainable directly, but, being the direction of every line joining two points, it is present in every relation between the beings of this world. In addition, the model can be used as a visualization assisting prayer (Krajewski 2019, p. 1014–1015).

The emerging picture is attractive, but mathematical research on the projective plane includes many more deep theorems than what was used in the model. They seem to have no theological parallels. If the geometrical model is seen as an adequate description of the philosophico-theological idea, philosophy and theology look meager compared to mathematics. If, on the other hand, one believes that it is philosophy and theology that deals with important problems of humankind and the world, then the mathematical model becomes no more than an illustration – a suggestive one, but meager and fundamentally irrelevant.

None of the presented examples of applications of mathematics to theology seems truly fundamental. If they help pondering some theological ideas, they are useful. However, they are only metaphorical and lack the features that make mathematical models so powerful in science, especially in physics, where they are creative; in particular, they suggest new concepts, and have predictive power, that is, enable us to foresee events or experimental results, and thereby check the model's adequacy. In theology, nothing so essential seems to be added by mathematical modeling.

6 Conclusion

Relations between mathematics and religion are seen by some as friendly, by others as antithetical – see the mottos to the present chapter. While modern mathematicians usually disregard the connection, some philosophers try to detect it. This can be done by affirming faith, as does Roy Clouser (1991), who argues that the right Biblical approach takes everything as dependent on God, therefore to attribute absolutely independent existence to numbers constitutes paganism. On the other hand, the criticism of Platonic assumptions can be done from an anti-religious position: "the actual Platonic infinity seems to invoke some version of the Greek–Hebraic divinity," states Brian Rotman (1993, p. 49). Any belief in the existence of mathematical objects leads to the "unstated theism – implicit and unacknowledged – of twentieth-century mathematical infinitism" (Rotman 1993, p. 157). This, one could add, seems to be a major reason for the attractiveness of mathematics. According to Vladislav Shaposhnikov, at the turn of the twentieth century "some key mathematicians of the period made an attempt (perhaps partially unconscious) to substitute pure mathematics for departed theology" (Shaposhnikov 2016, p. 48).

Up to modern times, the relation of mathematics to religion was close. According to Ivor Grattan-Guinness, “the main contexts lay in numerology, sacred geometry, and mechanics” (Grattan-Guinness 2011, p. 144). They are now of historical interest only. With the advent of modernity, the interplay of mathematics and theology continued. Ultimately, modernizing did mean secularization. Yet the process of getting to the present fundamentally irreligious science, including mathematics, was long and slow.

The examples mentioned in this chapter do not provide an indisputable proof that religion was indispensable for mathematical developments or that mathematical modeling was essential for theology. There is no doubt, however, that religious needs and motivations played a role in the development of mathematics, and mathematical metaphors helped theologians. Perhaps to have genuine mathematical models of divine matters another mathematics, still to be developed, is needed?

Is mathematics still connected to religion(s)? The present-day answer depends on how open is one’s attitude to the realm of the religious. For those sensitive enough to religion the connection can be perceived.

In addition to possible mutual influences, there exists another method of relating mathematics to theology. It seems that in both disciplines a central role is played by mystery: the more we know the more we realize the limitations of our knowledge. While such limitations are present in all sciences they are particularly prominent in apophatic theology and in mathematical incompleteness phenomena. Some of the main assumptions must be accepted for no other reason than faith. Again, this may be true about knowledge in every field because we often function within a paradigm, chosen without sufficient reasons – which does not mean irrationally. Yet mathematics and theology seem to be more arbitrary than other fields, which is due to the fact that both are more distant from the empirical world than are the other disciplines. Mathematics and theology are controlled by internal consistency requirements, fruitfulness, esthetic values, intuitive appeal. Even though intuition is shaped by empirical and cultural factors, they do not fully determine mathematical or theological assumptions. Positions in the philosophy of mathematics differ as widely as they do in theology.

Acknowledgment

The author is grateful to Andrew Aberdein for several suggestions that have helped correct and improve this chapter.

References

Abbott, Edwin A. (1952) *Flatland*. Dover, New York. Original ed. 1884.

Achtner, Wolfgang (2016) From religion to dialectics and mathematics: Schleiermacher's philosophical-theological contribution to the development of modern tensor calculus in Graßmann's Ausdehnungslehre. *Studies in Logic, Grammar and Rhetoric* 44 (57), 111-131.

Aigner, Martin and Günter Ziegler (2001) *Proofs from THE BOOK*. Second ed. Springer-Verlag, Berlin, New York.

Albertson, David (2014) *Mathematical Theologies. Nicholas of Cusa and the Legacy of Thierry of Chartres*, New York: Oxford University Press 2014.

Baldwin, John (2004) Review of Wolfram 2002. *The Bulletin of Symbolic Logic* 10 (2004), 112-114.

Barrow, John (2000) *The Book of Nothing. Vacuums, Voids, and the Latest Ideas about the Origin of the Universe*, Vintage Books.

Baum, Robert J. (ed) (1973) *Philosophy and Mathematics. From Plato to the Present.* San Francisco, Freeman, Cooper Co.

Benacerraf, Paul and Hilary Putnam (eds) (1983) *Philosophy of mathematics. Selected readings*. Second ed., Cambridge University Press.

Boethius (1897) *The Consolation of Philosophy*. Translated into English Prose and Verse by H.R. James. Elliot Stock, London. www.gutenberg.org/files/14328/14328-h/14328-h.htm

du Bois-Reymond, Emil (1870) Leibnizische Gedanken in der neueren Naturwissenschaft. In vol. 1 of du Bois-Reymond's collected works, Google Books: https://www.google.com/books/edition/Reden_von_Emil_Du_Bois_Reymond_v_1/SSkJAAAAIAAJ?hl=en&gbpv=0.

Borel, Emile (1972) *Œvres de Emile Borel*. Centre National de la Recherche Scientifique, Paris.

Bradley, James (ed) (2011) "Theology and Mathematics," special issue of *Theology and Science* vol. 9, No 1.

Bradley, James and Russell Howell (2011) *Mathematics through the Eyes of Faith*. HarperOne, New York.

Breger, Herbert (2005) God and Mathematics in Leibniz's Thought. In Koetsier & Bergmans (2005), 485–498.

Buber, Martin (1937) *I and Thou*. Transl. by Ronald Gregor Smith of 1923 *Ich und Du*. Clark, Edinburgh.

Byers, William (2007) *How Mathematicians Think. Using Ambiguity, Contradiction, and Paradox to Create Mathematics*. Princeton University Press.

Cantor, Georg (1932) *Gesammelte Abhandlugen mathematischen und philosophischen Inhalts*. Ed. by E. Zermelo, Verlag Julius Springer, Berlin; reprinted Springer, Berlin 1980.

Carnap, Rudolf (1930) The logicist foundations of mathematics. In: Benacerraf and Putnam 1983, 41-52.

Cohen, Daniel J. (2007). *Equations from God. Pure Mathematics and Victorian Faith*, Baltimore: The John Hopkins University Press.

Counet, J.-M. (2005) Mathematics and the divine in Nicholas of Cusa. In: Koetsier and Bergmans (ed.) 2005, 273-290.

Chase, Gene B. (1996) How Has Christian Theology Furthered Mathematics? Chapter 8 in: Van der Mee, Jitse (ed.) *The Role of Beliefs in Mathematics and the Natural Sciences* (Facets of Faith & Science, Vol II), Lanham, MD, University Press of America.

Clouser, Roy A. (1991) *The Myth of Religious Neutrality*. University of Notre Dame Press.

van Dalen, Dirk (1999) *Mystic, Geometer, and Intuitionist: The Life of L. E. J. Brouwer*. Oxford University Press 1999.

Dantzig, Tobias (1954) *Number. The Language of Science*, Doubleday-Anchor, Garden City.

Dauben, Joseph (1977) Georg Cantor and Pope Leo XIII: Mathematics, Theology, and the Infinite, *Journal of the History of Ideas* 38, No. 1, 1977, 85-108

Dauben, Joseph W. (1990) *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Princeton University Press.

Davis, Philip J., Reuben Hersh, and Elena A. Marchisotto (1995) *The Mathematical Experience*. Birkhäuser, Boston, Basel, Berlin.

Diller, Jeanine and Asa Kasher (eds) (2013) *Models of God and Alternative Ultimate Realities*. Springer, Dodrecht.

von Fritz, Kurt (1945) The discovery of incommensurability by Hippasus of Metapontum, *Annals of Mathematics*, 46(2), 242–264.

von Fritz, Kurt (1981) Pythagoras of Samos. In: *Dictionary of Scientific Biography*, vol. 11, ed. by A. Pitcairn and B. Rusch (gen. ed. C. S. Gillispie). Ch. Scribner's Sons, New York, 219–225.

Graham, Loren & Jean-Michel Kantor (2009) *Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity*. Harvard University Press.

Graham, Loren (2011) The Power of Names. *Theology and Science* vol. 9, nr 1 (2011), 157-164.

Grattan-Guinness, Ivor (2009) *Roots of Learning: Highways, Pathways and Byways in the History of Mathematics*. John Hopkins University Press, Baltimore.

Heller, Michał & Hugh Woodin (eds) (2011) *Infinity. New Research Frontiers*. Cambridge University Press.

Hersh, Reuben (1991) Mathematics has a front and a back. *Synthese* 88, 127-133.

Hersh, R. (ed.) (2006) *18 Unconventional Essays on the Nature of Mathematics*. Springer, New York et al.

Hilbert, David (1926) Über das Unendliche, *Mathematische Annalen* 95, 161-190. Transl. On the infinite, in Jean van Heijenoort (ed.) *From Frege To Gödel: A Source Book in Mathematical Logic, 1879–1931*. Harvard University Press 1967; and in Benacerraf and Putnam (1983), 183-201.

Kanigel, Robert (1992) *The Man Who Knew Infinity: A Life of the Genius Ramanujan*. Washington Square Press, Little Brown.

Koetsier, Teun and Luc Bergmans (eds) (2005) *Mathematics and the Divine: A Historical Study*. Elsevier.

Krajewski, Stanisław (2011) *Czy matematyka jest nauką humanistyczną?* [*Is Mathematics a Humane Science?* In Polish], Copernicus Center Press, Cracow.

Krajewski, Stanisław (2016) Theological Metaphors in Mathematics. *Studies in Logic, Grammar and Rhetoric* 44 (57), 13-30.

Krajewski, Stanisław (2019) Mathematical Models in Theology. A Buber-inspired Model of God and its Application to "Shema Israel". *Journal of Applied Logics* 6(6), 2019, 1007–1020.

Krajewski, Stanisław and Kazimierz Trzęsicki (eds) (2016) Issue „Theology in Mathematics?", *Studies in Logic, Grammar and Rhetoric* 44 (57).

Liber XXIV philosophorum. www.artandpopularculture.com/Liber_XXIV_philosophorum. French ed. by F. Hudry, Millon, Grenoble 1989.

Lawrence, Snezana and Mark McCartney (eds) (2015) *Mathematicians and Their Gods: Interactions between mathematics and religious beliefs*. Oxford University Press.

Lewis, Albert (2011) The Divine Truth of Mathematics and the Origins of Linear Algebra. *Theology and Science* vol. 9, nr 1 (2011), 109-120.

Menzel, Christopher (2018) The Argument from Collections. In: Walls, L. and Dougherty, T. (eds.) (2018), *Two Dozen (or so) Arguments for God: The Plantinga Project*, Oxford University Press, 29-58.

Miller, James B. (1998) From the Garden to Gauss: Mathematics as Theological Metaphor". AAAS Dialogue on Science, Ethics, and Religion.

https://web.archive.org/web/19990221051548/https://www.aaas.org/spp/dspp/dbsr/RESOURCE/miller/GAUSS4.htm

Moore, Gregory H. (1982) *Zermelo's Axiom of Choice. Its Origins, Development, and Influence*. Springer-Verlag, New York, Heidelberg, Berlin.

Mueller, I. (2005) Mathematics and the divine in Plato. In: Koetsier and Bergmans (eds) 2005, 99-121.

Netz, Reviel (2005) The Pythagoreans. In: Koetsier and Bergmans 2005, 77-98.

Nicholas of Cusa (1985) *On Learned Ignorance*, transl. by Jasper Hopkins. The Arthur J. Banning Press, Minneapolis, Minnesota.

Nicholas Of Cusa (1996) *Idiota De Mente (The Layman on Mind)*, transl. by Jasper Hopkins. The Arthur J. Banning Press, Minneapolis, Minnesota.

Nicomachus (1926) *Introduction to Arithmetic*, transl. by M. L. Dodge, with studies by Frank Egelston Robbins and Louis Charles Karpinski. The Macmillan Company, New York. https://babel.hathitrust.org/cgi/pt?id=mdp.39015005675411&view=1up&seq=9

Oppy, Graham (2011) God and Infinity: Directions for Future Research. In: Heller and Woodin 2011, 233-254.

de Pater, Cornelis (2005) An Ocean of Truth. In: Koetsier and Bergmans 2005, 459-484.

Plantinga, Alvin (2007) Two dozen (or so) theistic arguments. In: Dean-Peter Baker (ed), *Alvin Plantinga*, Cambridge University Press, 203–227.

Post, John F. (1974) New Foundations for Philosophical Theology, Quine with God. *The Journal of Philosophy* 7, 736-748.

Poythress, Vern S (2015). *Redeeming Mathematics. A God-Centered Approach*, Crossway, Wheaton, IL.

Reid, Constance (1996) *Hilbert*. Springer Verlag, Berlin.

Remmert, Volker R. (2005) Galileo, God and Mathematics. In: Koetsier and Bergmans 2005, 347-360.

Richards, Joan L. (1992) God, Truth, and Mathematics in Nineteenth Century England. In: M. J. Nye et al. (eds.), *The Invention of Physical Science*, Kluwer Academic Publishers, 51-78.

Rogers, Leo (2015) Historical Epistemology: Professional Knowledge and Proto-Mathematics in Early Civilisations. In: *History and Epistemology in Mathematics Education* Proceedings of the Seventh European Summer University ESU 7. Barbin, E, Jankvist, U T, Kjeldsen T H (eds), Danish School of Education, Aarhus.

Rotman, Brian (1993) *The Ghost in Turing's Machine. Taking God Out of Mathematics and Putting the Body Back In*. Stanford University Press.

Russell, Bertrand (1924) Atoms in modern physics. In: *Essays on Language Mind and Matter 1919-26*, Allen and Unwin, London, 1988. https://archive.org/stream/in.ernet.dli.2015.273552/2015.273552.Bertrand-Russell_djvu.txt

Seidenberg, Abraham (1961) The ritual origin of geometry. *Archive for History of Exact Sciences* 1, 488–527.

Seidenberg, Abraham (1962) The ritual origin of counting. *Archive for History of Exact Sciences*, vol. 2, 1-40.

Seidenberg, Abraham (1978) The Ritual Origin of Mathematics. *Archive for History of Exact Sciences* 18, 301–342.

Shaposhnikov, Vladislav (2016) Theological Underpinnings of the Modern Philosophy of Mathematics. Part I: Mathematics Absolutized, Part II: The Quest for Autonomous Foundations. *Studies in Logic, Grammar and Rhetoric* 44, 31-54 and 147-168.

Smith, Wilfred Cantwell (1964) *The Meaning and End of Religion*. Mentor Books, New York.

Sriraman, Bharath (ed.) (2017) *Humanizing Mathematics and its Philosophy. Essays Celebrating the 90th Birthday of Reuben Hersh*. Birkhäuser, Cham.

Tapp, Christian (2011) Infinity in Mathematics and Theology. *Theology and Science* vol. 9, No 1 (2011), 91-100.

Tegmark, Max (2014) *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality*. Knopf.

Thiele, R (2005) Georg Cantor (1845–1918). In: Koetsier and Bergmans 2005, 523-548.

Waerden, Bartel L. van der (1983) *Geometry and Algebra in Ancient Civilizations*. Springer-Verlag, Berlin etc.

Wolfram, Stephen (2002) *A New Kind of Science*. Wolfram Media.